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ELEC 5410
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Project #3: Sampling and Reconstruction

1. Examine a sampled waveform:

(a) **Sample the waveform x(t) = 3 cos 3000πt + cos 800πt at a sampling frequency of 16 kHz for one second:**

```
t = [0:1/16000:1];
xc = 3*cos(3000*pi*t) + cos(800*pi*t);
```
**We will treat this as though it is a continuous-time signal, since it is sampled without aliasing.**

(b) **Plot the first 100 points of the signal with an appropriately labeled t axis. (Note that you should use *plot* for signals that represent continuous time but *stem* for signals that are to be thought of as discrete. Functions of t and ω are continuous.)**

Figure 1: First 100 Points of xc
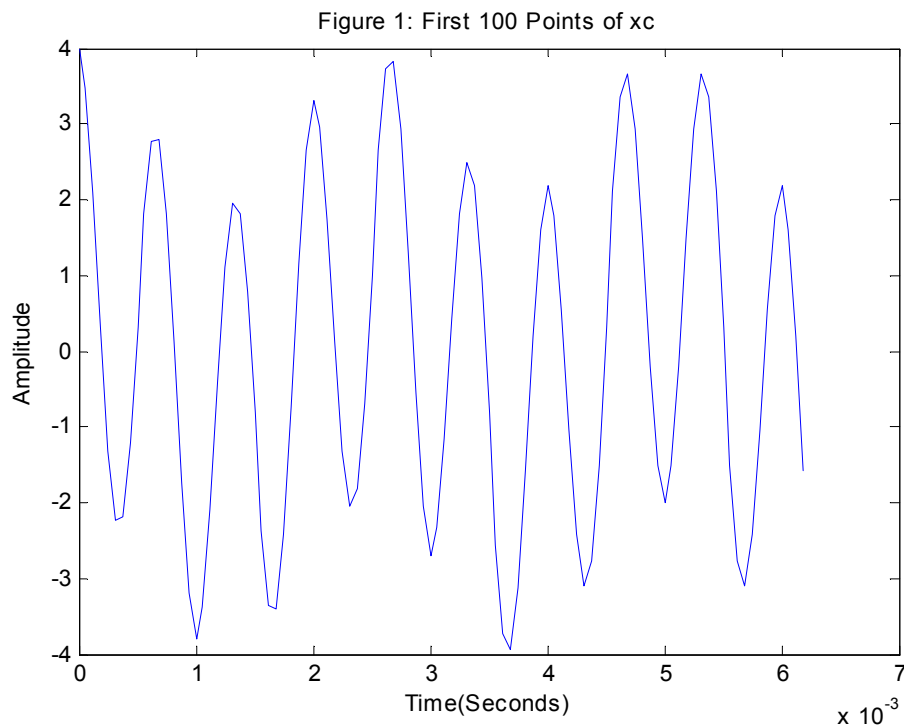
(c) **Plot the Fourier-transform magnitude of x(t) with the following command:**

```
plot([-8000:7999],fftshift(abs(fft(xc(1:2*8000)))))
xlabel('normalized frequency (\times 2\pi to give rad/s)')
```

Figure 2: FFT of xc sampled at 16kHz
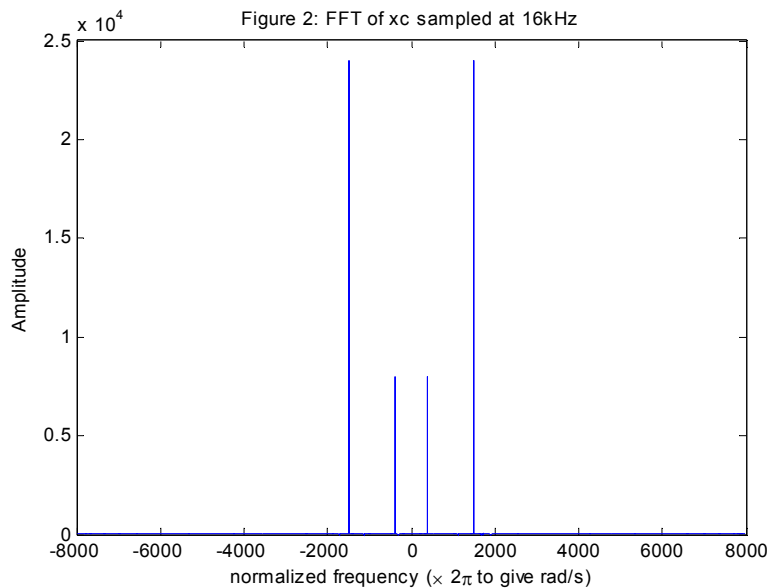
(d) Explain how this graph corresponds to the actual Fourier transform of x(t).
Figure 2 shows the Fourier-transform of the original signal. Because this is a digital Fourier
transform, the amplitude of the impulses is proportional to the samples and the frequencies are
accurate. The impulses are at 1500Hz and 400Hz, just as the original signal, and the deltas at
1500Hz are three times the amplitude of that of the 400Hz.

2. Now we will consider the effect of sampling on the original signal.
   **(a) Sample the waveform x(t) = 3 cos 3000πt + cos 800πt at a sampling frequency of 8 kHz for one
   second to obtain x[n], where t = nT.**
   **(b) Plot the resulting sequence using *stem* the first 100 points of the signal with an appropriately
   labeled n axis.**

Figure 3: First 100 Points of xc Sampled at 8kHz
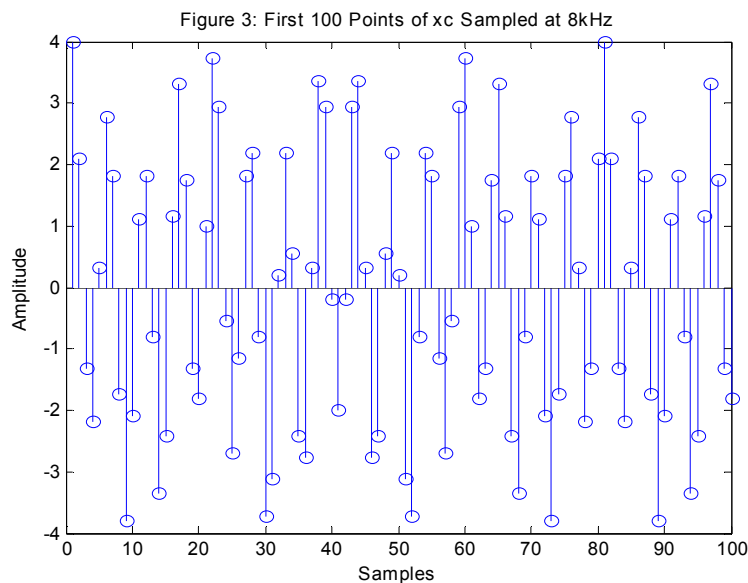
(c) Plot the DTFT magnitude:
```
plot([-4000:3999]/8000,fftshift(abs(fft(x(1:8000)))))
xlabel('normalized frequency (\times 2\pi to give rad/sample)')
```

Figure 4: DTFT of xc Sampled at 8kHz
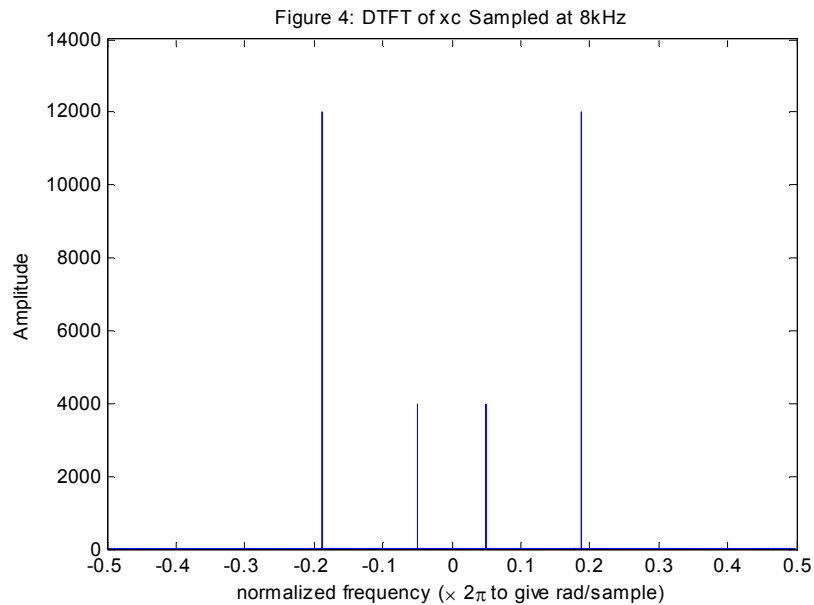
It may also help to plot three repeated copies of the spectrum and try to identify where the spectral components of each original spectrum are in that plot:
```
plot([-12000:11999]/8000,repmat(fftshift(abs(fft(x(1:8000)))),[1 3]))
xlabel('normalized frequency (\times 2\pi to give rad/sample)')
```

Figure 5: Repeated Copies of DTFT of xc Sampled at 8kHz
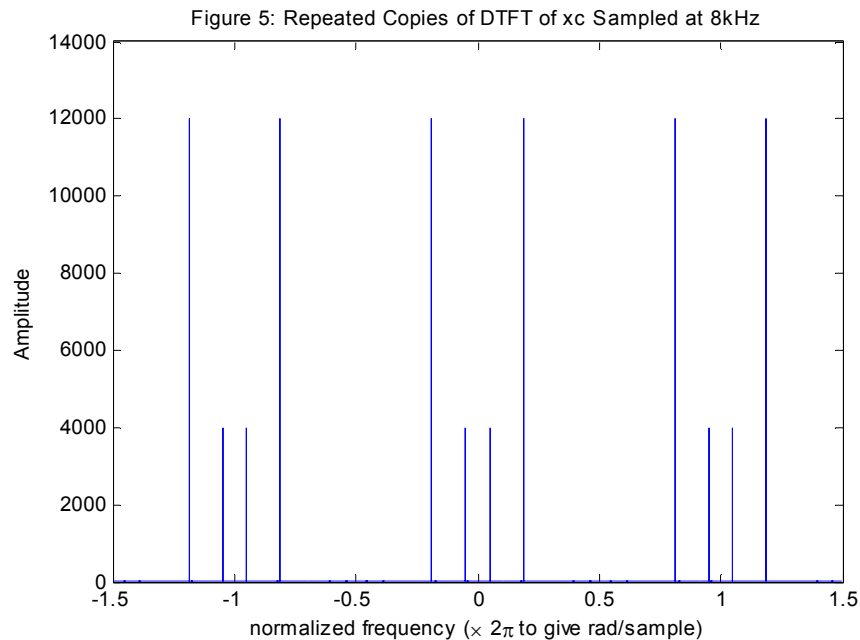
(d) Does aliasing occur? Explain.
No, aliasing does not occur since the sampling frequency is greater than twice the greatest frequency in the original signal (Greatest Frequency = 1500Hz) by Nyquist's theorem.

3. Now consider a different sampling rate:
  (a) Sample the waveform in #1 at a sampling frequency of 2 kHz for one second.
  (b) Determine how to plot the DTFT magnitude as in #2. You may also want to plot the repeated spectrum as above to better visualize the original spectrum within the repeated copies.

Figure 6: DTFT of xc Sampled at 2kHz

Figure 7: Repeated Copies of DTFT of xc Sampled at 2kHz
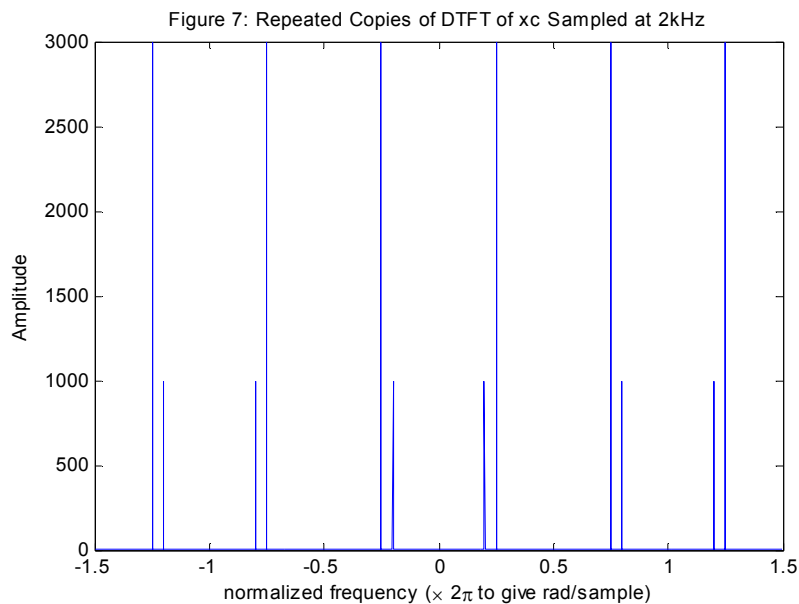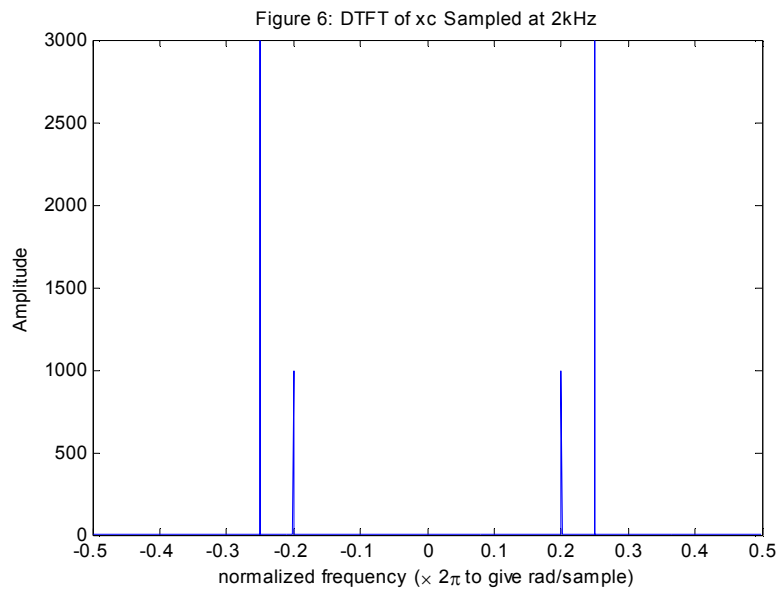
(c) Explain the differences between this plot and the one in #2 based on the properties of sampling.

Figure 6 shows the FFT of the original waveform sampled at such a rate as to introduce aliasing. By comparing the Figures 6 & 7 with Figures 4 & 5, we can easily see that both the amplitude and frequency are different. It is quite evident that the sampling rate is too slow and some information in the signal has been lost.

(d) Does aliasing occur?

Yes, aliasing does occur since 2kHz is less than the 3kHz frequency necessary to prohibit aliasing.

4. We will now attempt to reconstruct the signal in #1 from the sequence in #2 using zero-order hold.
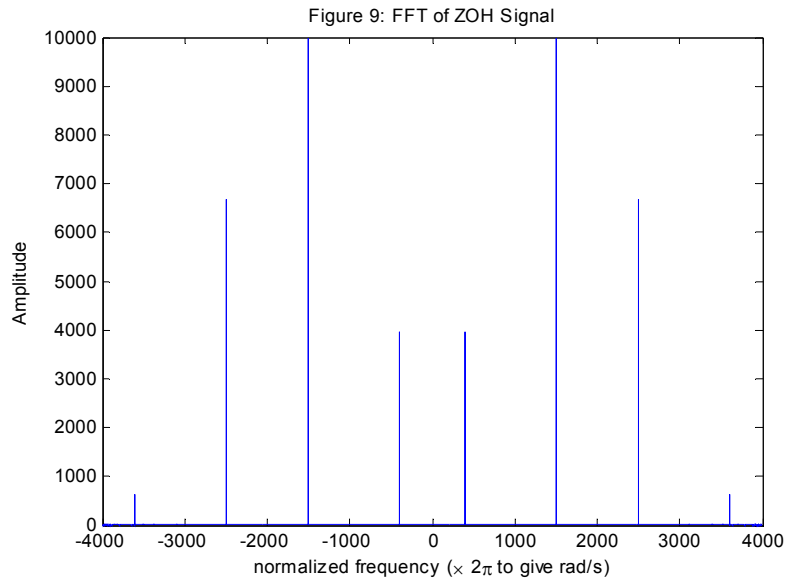
(a) Form a zero-order hold signal with 2 equal output samples per input sample. Plot the first 100 points considered as a continuous-time signal.

(b)

```
--------------------
xzoh=zeros(1,8000);
for i=1:2:8000
    xzoh(i)=x(i);
    xzoh(i+1)=x(i);
end
plot(xzoh(1:100));
--------------------
```


Figure 8: Zero Order Hold
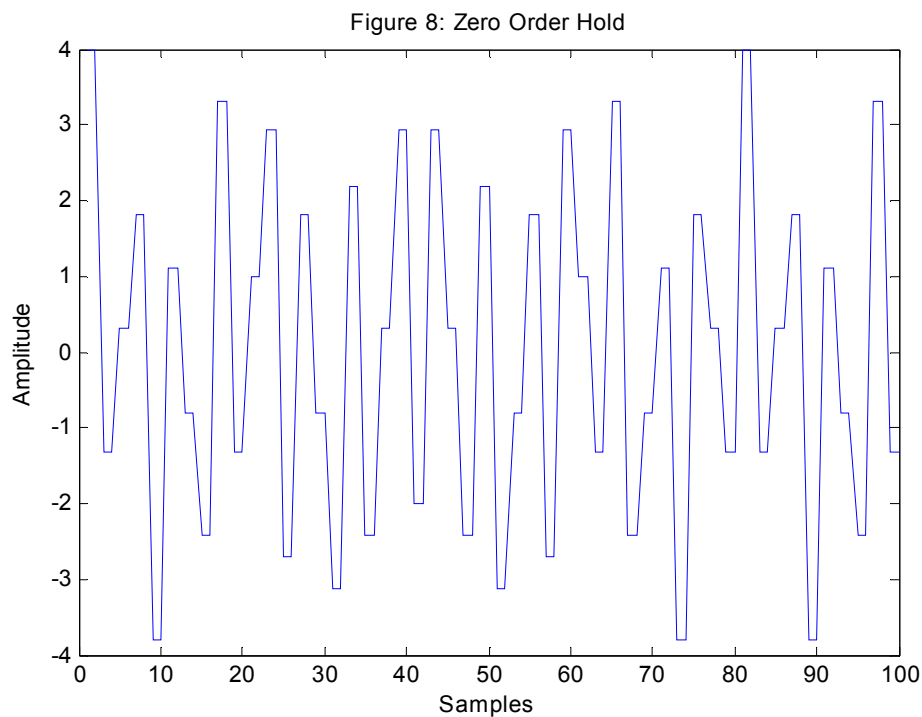
(c) Plot the FT magnitude using the syntax from #1, and explain what you observe.

Figure 9: FFT of ZOH Signal

We see what appears to be the original signal with higher frequency content from the sampling process as expected. It should be noted that the amplitude of the two impulse pairs seen as the original signal have an amplitude that does not necessarily correlate to the original amplitude or the sampling.
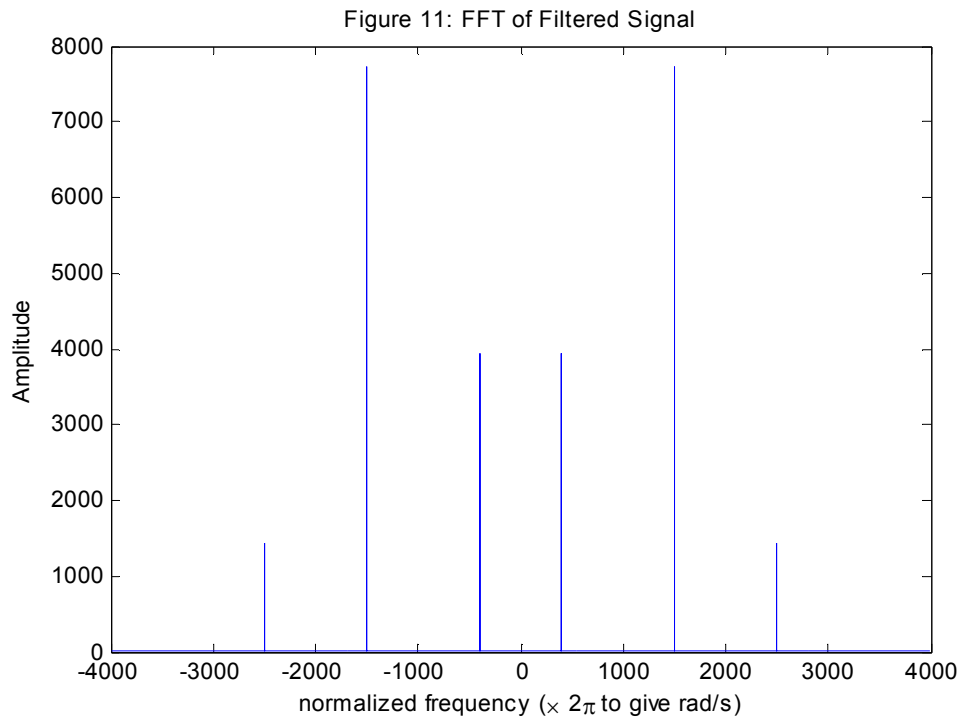
(d) Lowpass filter the ZOH signal as follows:
```
h = fir1(9,1/2);
xr = filter(h,1,xzoh);
```
(e) Plot the first 100 points considered as a continuous-time signal.

Figure 10: Low Pass Filter of the ZOH Signal
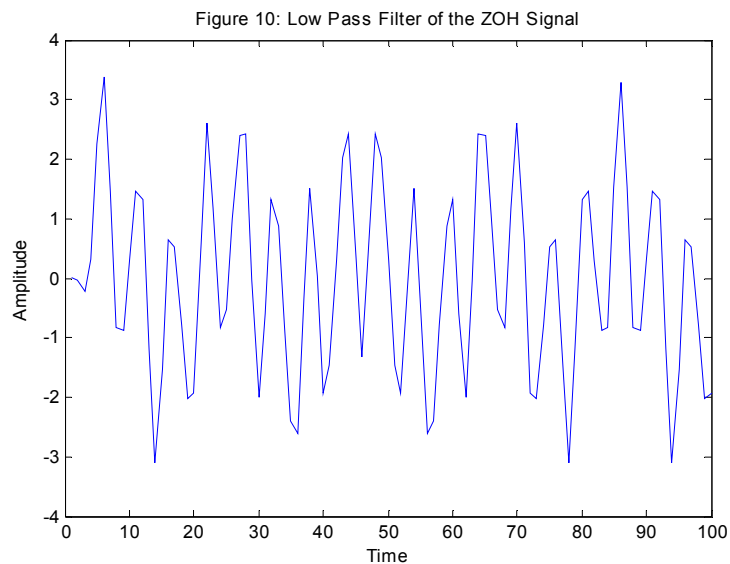
(f) Plot the FT magnitude of the result using the syntax from #1. Explain what you observe. Is the signal in #1 perfectly reconstructed? Justify your answer.

Figure 11: FFT of Filtered Signal

No, the filtered signal is not a perfect reconstruction of the original signal. Some of the frequencies are the same as in the original signal, but there is a slight loss in the amplitudes and two extra frequency components.

5. Play the sound from #1, the ZOH sound, and the reconstructed sound using:
`soundsc(x,16000)`
**Note any differences in the sounds. (You may need a set of headphones in the lab.)**
The original signal sounds like a high pitched beep. The ZOH sounds like a lower pitched beep with a separate higher tone at the same time. The filtered and reconstructed tone sounds like just a lower pitched beep.

6. 6410 and extra credit for 5410: Reconstruct the signal in #1 from the sequence in #2 using linear interpolation. (See pp. 70-71 in Mitra for discussion.) You must use the filter function to get maximum credit. Compare the FT magnitude to the ZOH interpolation and the original. Explain the differences.
```
----------
a=1:8001;
b=1:8001;
c= interp1(a,x,b);
plot(c(1:100));
--------
```

Figure 12: Linear Interpolation Reconstruction
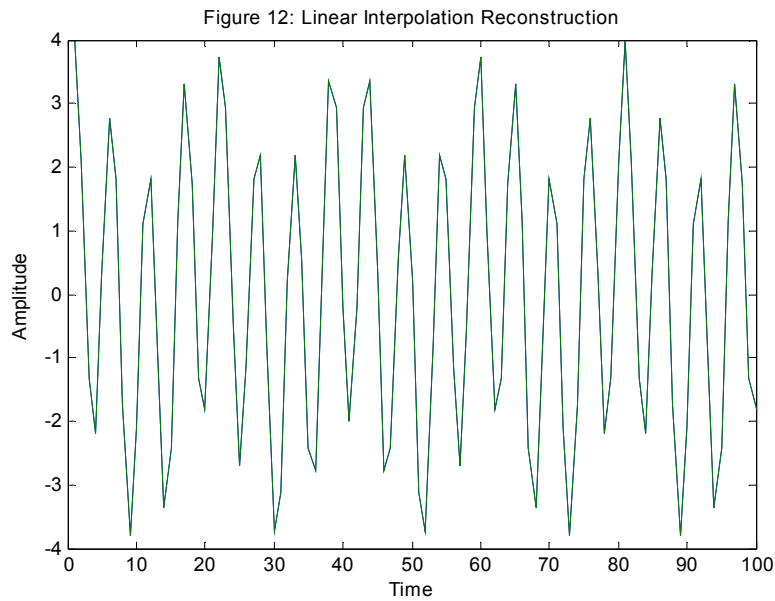

```
-----------
hint=fir1(9,1/2);
xint=filter(hint,1,c);
plot(xint(1:100))
plot([-4000:3999],fftshift(abs(fft(c(1:8000)))));
-----------
```

Figure 13: Filter Reconstruction

Figure 14: Linear Interpolation Reconstruction FFT
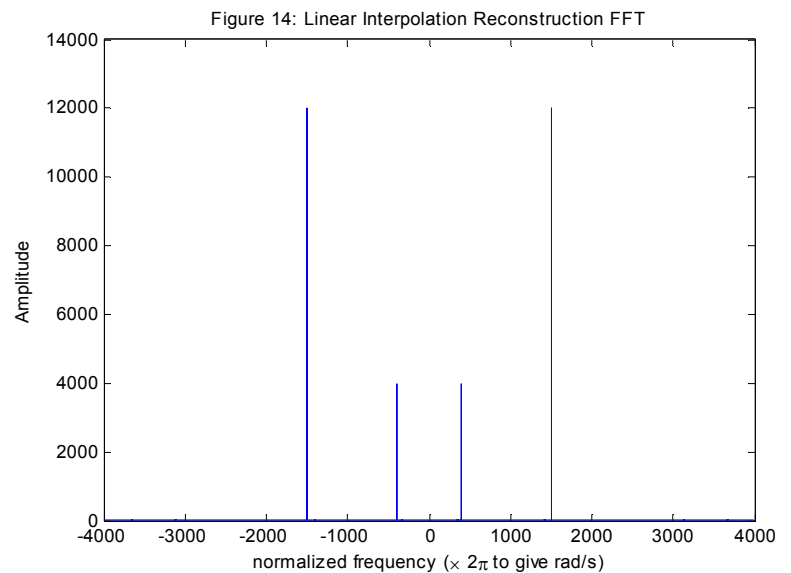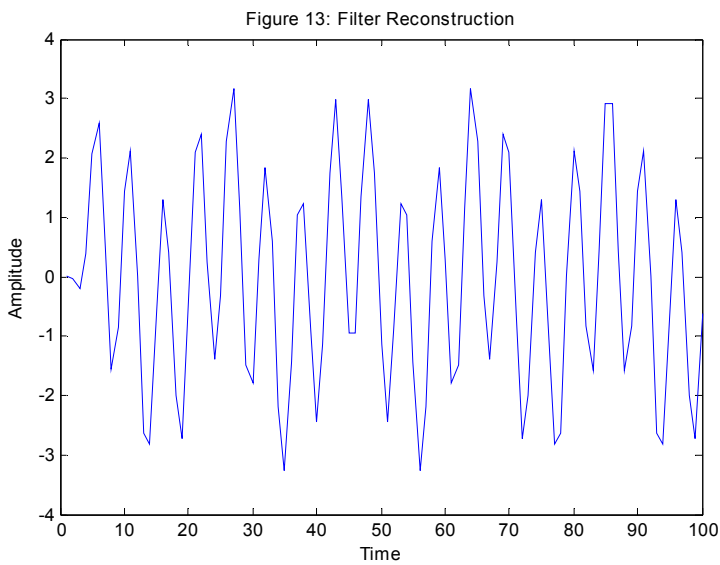
In the time domain, the signal looks similar to the ZOH filtered waveform, but in the frequency domain it matches the original signal surprisingly well, although the amplitude of the deltas does not seem to match perfectly.