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ELEC 5410
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Project #5: Z-transforms and Pole-Zero Response

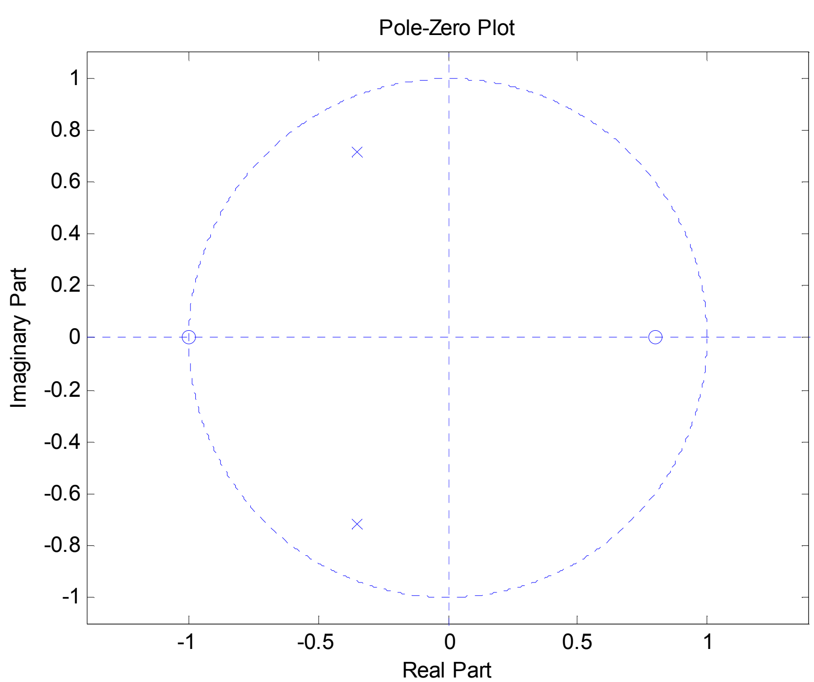
1. Consider a two-pole/two-zero system function:

$$H(z) = \frac{1 + 0.2z^{-1} - 0.8z^{-2}}{1 + 0.7z^{-1} + 0.64z^{-2}}$$

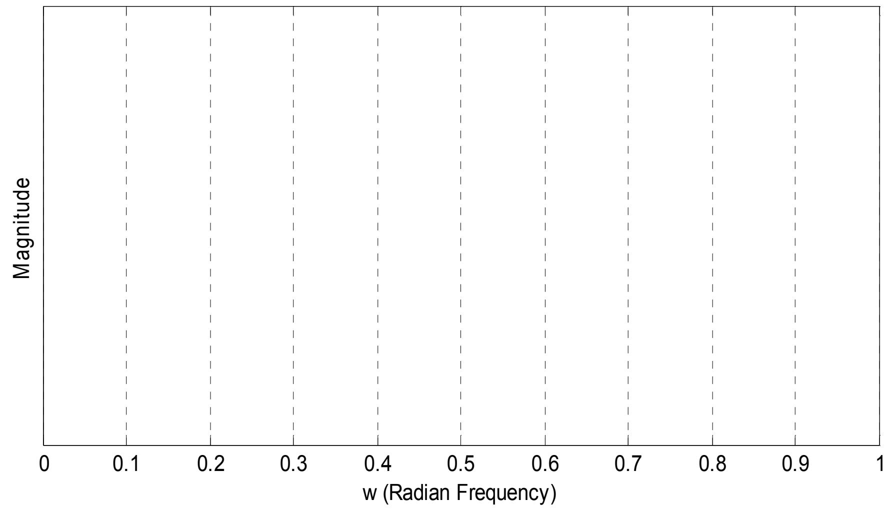
(a) Find the pole and zero locations of this system. (Hint: the functions `roots` and `poly` may be useful in this exercise.)

```
num=[1,0.2,-0.8]; den=[1,0.7,0.64];
zeros=roots(num)
poles=roots(den)
zeros =
   -1.0000
    0.8000
poles =
  -0.3500 + 0.7194i
  -0.3500 - 0.7194i
```

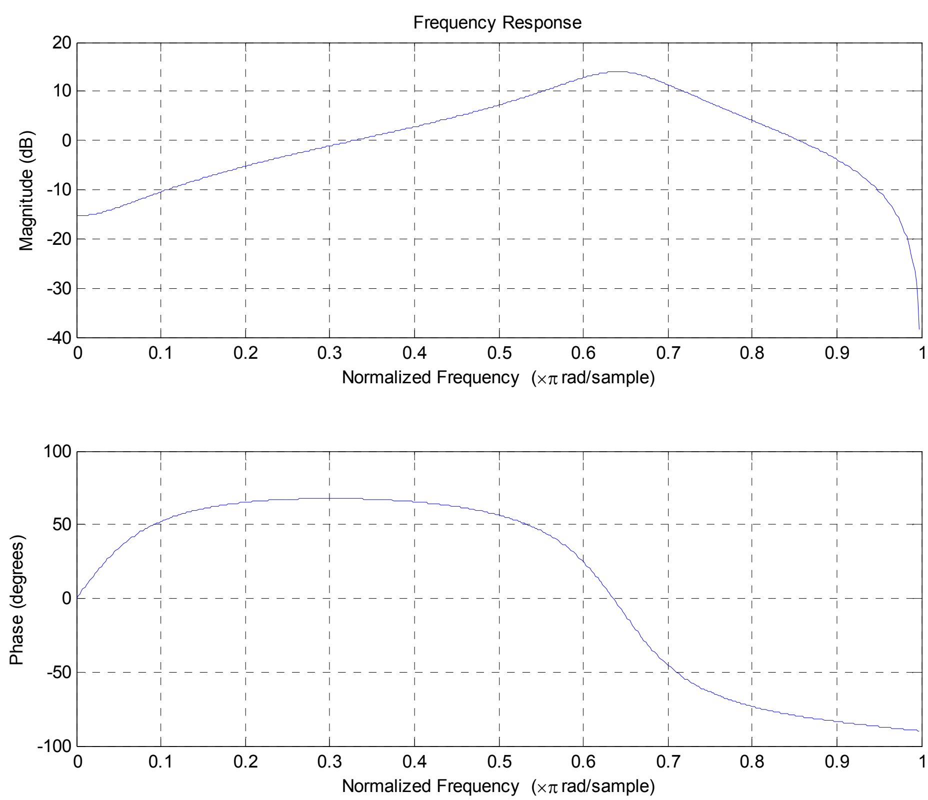
(b) Generate a pole-zero plot using `zplane`.



(c) Sketch the frequency response you expect to see by looking at the pole-zero plot.



(d) Display the frequency response with `freqz`. Note that your sketch will reflect magnitude response, while the MATLAB plot will show log-magnitude response.

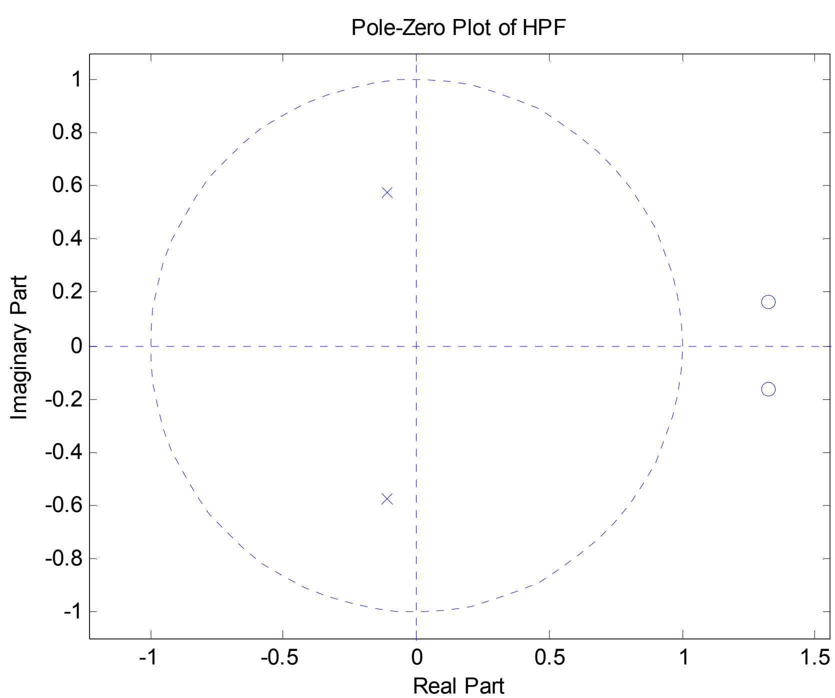


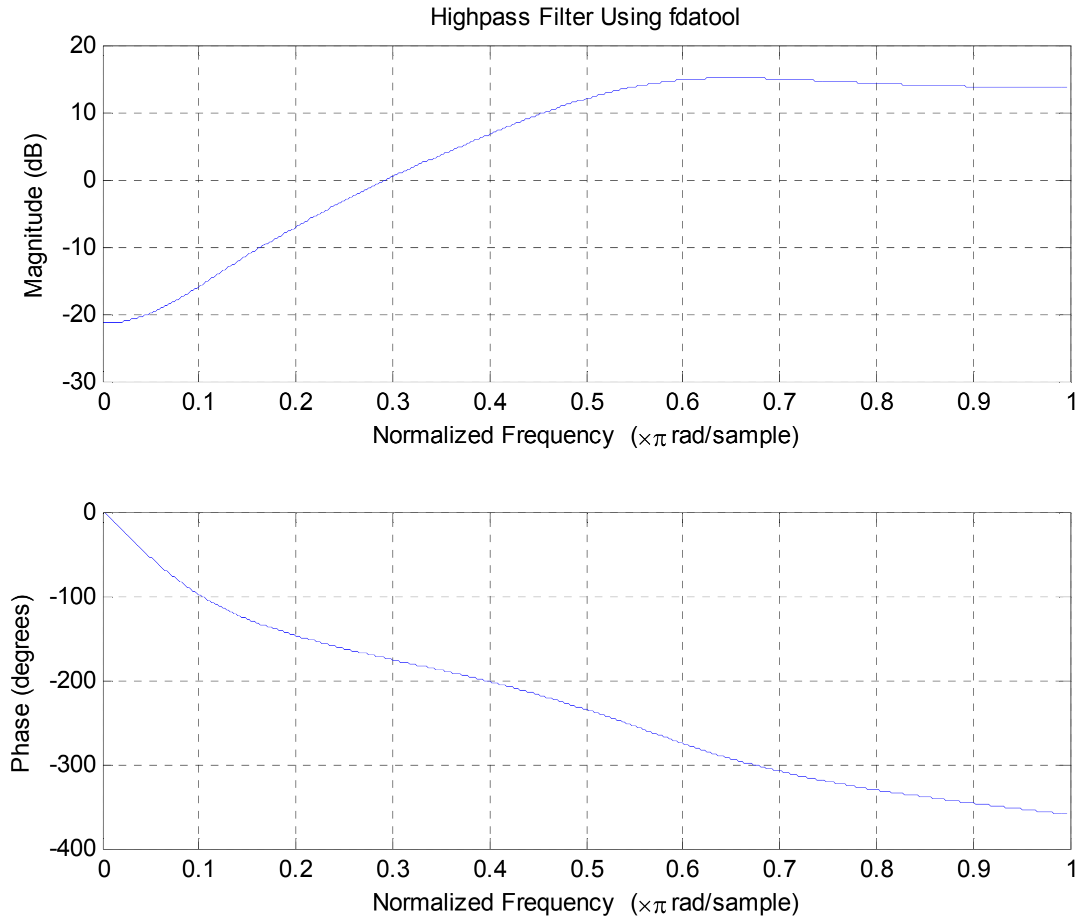
2. Run `fdatool`. Go to File->Import Filter from Workspace. Select Direct-Form 1 filter structure, and specify [1 0.2 -0.8] for the numerator and [1 0.7 0.64] for the denominator. Then click "Import Filter." Go to the pole-zero editor by clicking the appropriate icon on the left side. Using trial-and-error, determine the pole and zero locations of a system so that it implements the best highpass filter possible with cutoff π/2 using two poles and two zeros. ("Best" may be in the eye of the beholder to a degree.) Be sure to choose the poles and zeros so that the difference equation has real coefficients (complex-conjugate pole and zero locations). If you want complex-conjugate zeros, unselect "Conjugate", drag a zero off of the real axis, reselect "Conjugate", and then delete the unwanted zero that remains on the real axis. Select File->Export... and export the resulting coefficients to variables in MATLAB.

(a) Report the coefficients.

```
Num2 =
    1.0000   -2.6512    1.7851
Den2 =
    1.0000    0.2220    0.3439
```

(b) Provide pole-zero and magnitude response plots.





3. (a) Determine the difference equation defined by the filter you designed in the previous step.

$$\boldsymbol{H}(\boldsymbol{z}) = \frac{1 - 2.6512z^{-1} + 1.7851z^{-2}}{\mathbf{1} + 0.2\mathbf{22}\boldsymbol{z}^{-\mathbf{1}} + \mathbf{0}.3439z^{-2}} = \frac{Y(z)}{X(z)}$$

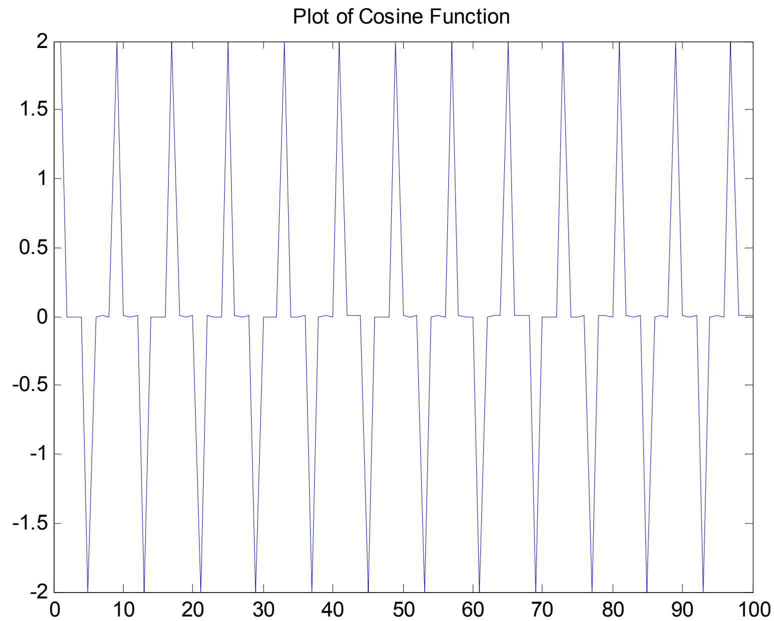
$$X(z)(1 - 2.6512z^{-1} + 1.7851z^{-2}) = Y(z)(\mathbf{1} + 0.2\mathbf{22}\boldsymbol{z}^{-\mathbf{1}} + \mathbf{0}.3439z^{-2})$$

$$x[n] - 2.6512x[n-1] + 1.7851x[n-2] = y[n] + 0.222y[n-1] + 0.3439y[n-2]$$

$$\boxed{y[n] = -0.222y[n-1] - 0.3439y[n-2] + x[n] - 2.6512x[n-1] + 1.7851x[n-2] \quad \blacksquare}$$

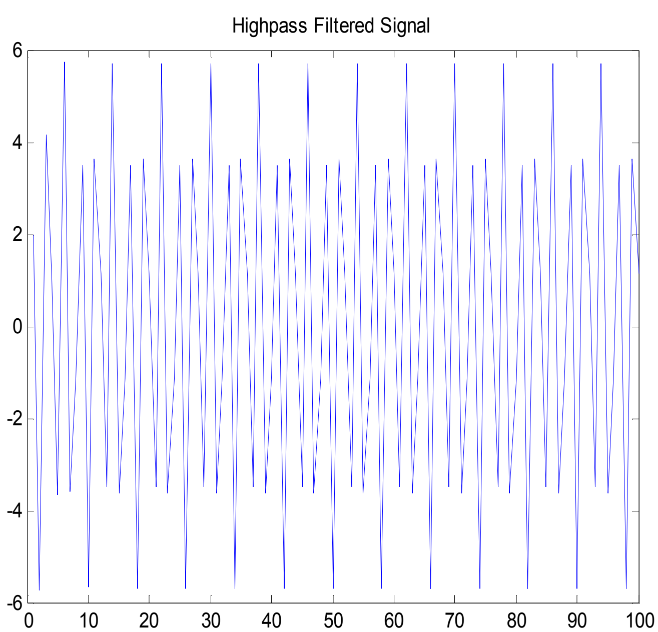
(b) Generate a signal x[n] = cos(0.25πn) + cos(0.75πn) of length 100.

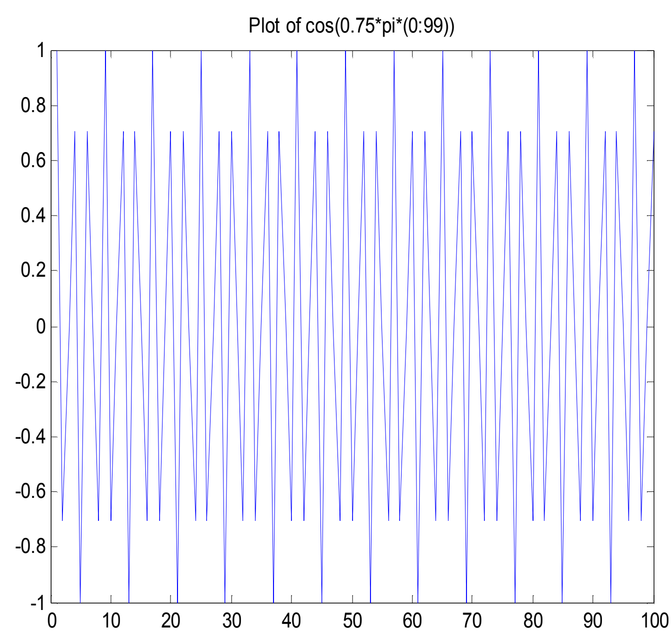
```
xn = cos(0.25*pi*(0:99)) + cos(0.75*pi*(0:99));
```



(c) Using the `filter` function, filter this signal with the filter you designed, and examine the result.

```
a=[1,0.222,0.3439]; % Denominator
b=[1,-2.6512,1.7851]; % Numerator
plot(filter(b,a,xn));
figure
plot(cos(0.75*pi*(0:99)));
```

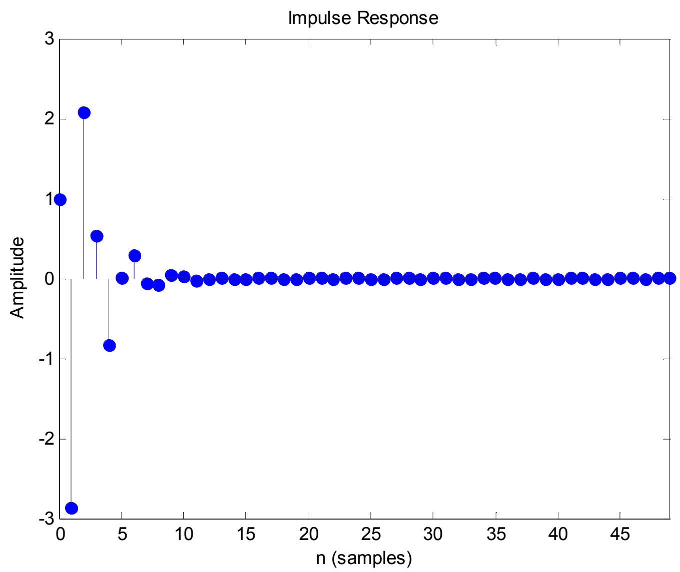




(d) Explain this result in terms of the frequency response of your filter.

The highpass filter seems to amplify the signal and filter the low frequency components. The result seems accurate when compared to the cos(0.75πn) plot, since the cos(0.25πn) part should be filtered out.

4. (a) Generate a 50-point impulse response from the difference equation using `impz`.



(b) Does this impulse response look like it would implement a highpass filter? (Think about a modulated lowpass filter impulse response. Or, does the impulse response appear to contain high frequencies and not low frequencies?) Explain.

This impulse response will indeed implement a highpass filter because the highest frequencies will have the greatest magnitude as indicated in the plot. A HPF impulse response should be similar to a sinc function, and the plot above seems to be half a sinc. The last 80% of the samples have nearly zero amplitudes.

5. Read in fanfare.au with `auread`. Filter this signal with the filter you designed and listen to the result with `sound`. Comment on the difference before and after filtering.

In the sound clip before the filter, one can hear the entire band, including the lower pitched instruments such as the tubas and trombones (low frequency components) whereas the clip after the filter sounds much higher and only the trumpets are higher pitched instruments can be heard, as if it has been highpass filtered. This effect makes sense because this describes the effect of a highpass filter.