Project #7: FIR Filter Design

1. Design a length-21 linear-phase FIR lowpass filter to approximate an ideal response with a passband edge of 0.25π . (Note that length-21 corresponds to a filter order of 20.) Design a filter using each of the following windows:

- rectangular (rectwin)
- triangular (triang)
- Hanning (hann)
- Hamming (hamming)
- Blackman (blackman)

Plot the impulse response, magnitude response, and zero-pole locations of the five filters. You may use fir1 to do the designs. (Use freqz and zplane for magnitude response and pole-zero plots.) Use subplot here and below to plot all three of these in the same row, and make them as small as you can without compromising legibility. Calculate frequency response with [h,w]=freqz(b,a); and then call semilogy (w, abs (h)) to see the magnitude response. This is the best way to get it to display in a subplot. Compare the characteristics of the magnitude response. NOTE: You will need to use a length-(21+2) window for Hann and Blackman windows and then pick out the middle 21 values, since these produce zeros on the end-points. Otherwise, your zero-pole locations will be in error.



о.з

0.25

0.2

0.15

0.1

0.05

0

-0.05 L

0.3

0.25

0.2

0.15

0.1

0.05

0

-0.05 L

о.

0.25

0.2

0.15

0.1

0.05

-0.05 C

10

15 20



10⁻⁷0

0.2

0.4

0.8

0.6

0 Real Part

Compare the characteristics of the magnitude response.

The rectangular window has the most passband ripple and contains the least amount of attenuation in the stopband. The triangular window has a much smoother roll-off into the stopband. It also has a relatively flat passband region and the amount of attenuation is a little higher than the rectangular filter in the stopband. The Hanning window has a greater amount of attenuation than either of the previous two windows. The Hamming window has a sharper roll-off and also has a more constant attenuation in the stopband than the Hanning filter, but offers a lower attenuation overall. The Blackman window has the highest stopband attenuation and seems to be the closest filter to ideal of the five cases tested.

2. Design a length-21 linear-phase FIR lowpass filter using a Kaiser window with β = 4, 6, & 9. Plot the impulse response, magnitude response, and zero-pole locations. Compare the characteristics of the magnitude response with each other and with the filters from the first exercise. How does the trade-off of transition bandwidth and overshoot vary with β ?







Compare the characteristics of the magnitude response with each other and with the filters from the first exercise. How does the trade-off of transition bandwidth and overshoot vary with β ? When $\beta = 9$, the Kaiser filter has a better attenuation than the Blackman filter and thus has a better stopband attenuation than all of the other filters. When $\beta = 6$, the Kaiser filter has a worse attenuation than the Blackman filter and Hanning filter but better than the triangular and rectangular filters, although the transition is sharper than for $\beta = 9$. When $\beta = 4$, the stopband attenuation is much worse than the Hamming and Blackman but better than the rectangular and triangular filters, but the transition to stopband is much sharper than $\beta = 6$ and 9. Therefore, as you increase β , the transition bandwidth and overshoot become larger.

3. Use frequency sampling to design a length-21 FIR lowpass filter with cutoff 0.25π (Do not use fir2). Determine the desired DFT sample values from the specified cutoff (provide a listing or MATLAB command), and then take the inverse FFT (ifft). Keep in mind that the DFT coefficients must be symmetric to yield a real impulse response. For example, a cutoff of 0.5π and a length of 9 would yield a DFT coefficient vector $[1\ 1\ 1\ 0\ 0\ 0\ 1\ 1]$ corresponding to the frequency values $2*pi*[0\ 1/9\ 2/9\ 3/9\ 4/9\ 5/9\ 6/9\ 7/9\ 8/9]$. The k=0 value is the first entry in the vector. Note that the impulse response must be circularly shifted so the origin is in the middle instead of at 0. Plot the impulse response, magnitude response to the other designs.

Determine the desired DFT sample values from the specified cutoff (provide a listing or MATLAB command)...

x=[1 1 1 zeros(1,16) 1 1];



Compare the characteristics of the magnitude response to the other designs.

The stopband attenuation of this lowpass FIR is slightly worse than the Parks-McClellan filter (in Part 4), and much worse than all of the other filters. This is because the attenuation is much lower than those filters and still lower than the Parks-McClellan filter at about 68.2 dB. It also has a sharp roll-off in the transition from passband to stopband.

4. Design an optimal 21-point lowpass FIR filter using the Parks-McClellan algorithm (firpm). Use a passband cutoff of 0.23π and a stopband cutoff of 0.27π . Plot the impulse response, magnitude response, and zero-pole locations. Compare the characteristics of the magnitude response to the other designs.



Compare the characteristics of the magnitude response to the other designs.

The above Parks-McClellan lowpass FIR filter has the second lowest stopband attenuation (at 69.7 dB) of all the filters, second to the FIR LPF designed in part 3 above. The passband region is not smooth and the roll-off is very sharp compared to all of the other filters.

5. Design an 11th-order elliptic lowpass filter with a passband ripple of 1 dB, a passband cutoff of 0.24π , a stopband cutoff of 0.26π , and the largest possible stopband attenuation (Use ellipord by trial and error to determine this). Compare the magnitude response to the Parks-McClellan filter. Note that an 11th-order IIR filter requires approximately the same number of multiplies per output point as a length-21 FIR filter because it has both pole and zero coefficients.



Compare the magnitude response to the Parks-McClellan filter.

The largest possible stopband attenuation is 130 dB for an 11^{th} order elliptic LPF with passband ripple of 1 dB, passband cutoff of 0.19π , and stopband cutoff of 0.21π . We can see that the stopband attenuation for the Parks-McClellan filter is much less than this. By comparing 130 dB of the elliptic filter to 69.7 dB of the Parks-McClellan, the elliptic filter has much better stopband attenuation by several orders of magnitude than the Parks-McClellan filter.

6. **6410 and extra credit for 5410:** Read in the file doorbell.au using auread. Using the fft function, determine the frequencies of the upper and lower tone of the doorbell. Using trial-and-error, design a length-21 filter using any of the FIR techniques above such that the lower tone is attenuated no more than 3 dB while the upper tone is attenuated as much as possible. Describe how you designed your filter.



Using the fft function, determine the frequencies of the upper and lower tone of the doorbell. From the figure, the upper frequency is 712.4 Hz ($\omega_s = 4467 \text{ rad/s} = 1425\pi \text{ rad/s}$) and the lower frequency is 567.4Hz ($\omega_p = 3565 \text{ rad/s} = 1135\pi \text{ rad/s}$). Therefore, for the FIR filter I designed, the passband cutoff frequency is 567.4Hz ($\omega_p = 3565 \text{ rad/s} = 1135\pi \text{ rad/s}$) and the stopband cutoff frequency is 712.4 Hz ($\omega_s = 4467 \text{ rad/s} = 1425\pi \text{ rad/s}$).

Describe how you designed your filter.

I decided to use the Hamming filter because it had a constant stopband attenuation which was extremely low. I want the 3 dB point to occur at 567.2 Hz for the LPF. So I normalized the frequency and divided 567.2 Hz by the sampling frequency, $F_s = 8000$ Hz. Then I multiply that value by 2π in order to convert to radians. So I made the filter with the passband attenuation at that value of 0.1418 π . The passband attenuation is defined as the 3 dB point. So that fills the constraint of have the lower tone attenuated by 3 dB and the upper tone attenuated much more.