

ECE 6390 Satellite Communication and Navigation Systems

Fall 2011 Homework Assignment 1

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Assignment

Find a satellite in an elliptical earth orbit ($e > .1$) and simulate its sky track from a vantage point in Atlanta (or elsewhere, if not visible from Atlanta). Graph the sky track and write a paragraph about the application of this satellite. You may not use Tundra or Molniya orbits (already discussed in class). You may work alone or in partners.

1 Solution

For the vantage point, I chose the Georgia Tech Van Leer building in Atlanta (33.7758 latitude, -84.39738 longitude). A suitable satellite for the assignment is the European Space Operations Centre's (ESOC's) INTERNATIONAL Gamma-Ray Astrophysics Laboratory, INTEGRAL, or just Integral. At its initial launch on October 17, 2002, it had an elliptical orbit with an eccentricity of 0.897 ($e \gg 0.1$). Its perigee height was 2,284 km, apogee 152,812 km, inclination 52.1 degrees, and period is 4,032.86 minutes. [1] However, due to orbit degradation, the satellite has now shifted to the following parameters obtained from Celestrak's TLE data.

INTEGRAL TLE

```
1 27540U 02048A 11253.62500000 -.00000555 00000-0 10000-3 0 774
2 27540 74.1108 309.0463 8918072 250.2922 355.0173 0.33417304 7021
```

Epoch (UTC): 15:00:00, Saturday, September 10, 2011

Eccentricity: 0.8918072

Inclination: 74.1108°

Perigee height: 3112 km

Apogee height: 159565 km

Right Ascension of ascending node: 309.0463°

Argument of perigee: 250.2922°

Revolutions per day: 0.33417304

Mean anomaly at epoch: 355.0173°

Orbit number at epoch: 702

[2] [3]

To be more accurate and current on my simulation, as well as potentially having the ability to confirm my own simulation through other simulators that use current data, I decided to use this data for my calculations.

Here is an artist's rendition of Integral. [4]



1.1 Simulation

For the simulation, I used the following equations for the 2D orbit:

$$\ddot{r} = r\dot{\theta}^2 - \frac{GM_P}{r^2}, \quad \ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r}$$

These are the basic system of differential equations that describe orbital motion around a single planet in polar coordinates. To specify the state of the orbit, all that is needed is a current trajectory (r, θ) and the velocity at that point. Since the velocities were unknown, I just iterated through V_θ until the period was close enough to the actual period and assumed that the satellite was at perigee so the V_r would be 0.

To translate these 2 dimensional coordinates into 3-space, I used the following equations.

$$x' = r\cos(\theta), \quad y' = r\sin(\theta), \quad z' = 0$$

Elevation (z') is assumed to be zero at this point. Inclination is achieved via rotation about the y-axis:

$$x = x' \cos \alpha - z' \sin \alpha, \quad y = y', \quad z = x' \sin \alpha + z' \cos \alpha, \quad \alpha = \text{inclination angle (degrees)}$$

Finally, use the following equations to convert the 3-space coordinates into latitude and longitude.

$$R = \sqrt{x^2 + y^2 + z^2}$$

$$\text{Latitude} = \sin^{-1}\left(\frac{z}{R}\right), \quad \text{Longitude} = \begin{cases} \tan^{-1}\left(\frac{y}{x}\right), & x > 0 \\ \pi + \tan^{-1}\left(\frac{y}{x}\right), & x < 0 \end{cases}$$

Discretizing the differential equations and coding the other equations in MATLAB, I was able to create a plot of the 2 dimensional orbit:

Orbit ($r=158813\text{km}$, $\theta=6^\circ$, $V_r=0.0\text{km/s}$, $V_\theta=0.7\text{km/s}$) Eccentricity, $e=0.888$, Period, $T=71.82\text{hr}$

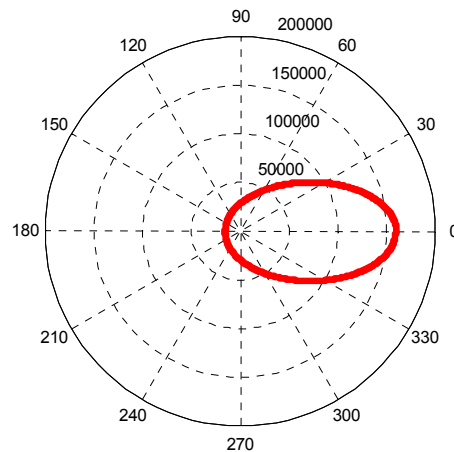


Figure 1: 2D Orbit of Integral Satellite

For calculating the look angles, several assumptions needed to be made. First, we assume the Earth station is directly below perigee (or apogee) so that we may assume the radial component of velocity to be 0. Second, we must assume that there are no orbital perturbations. Below is a look angles plot of elevation angle with respect to the azimuth (look) angle (Figure 2) and a straight up view from the Earth station vantage point (Figure 3).

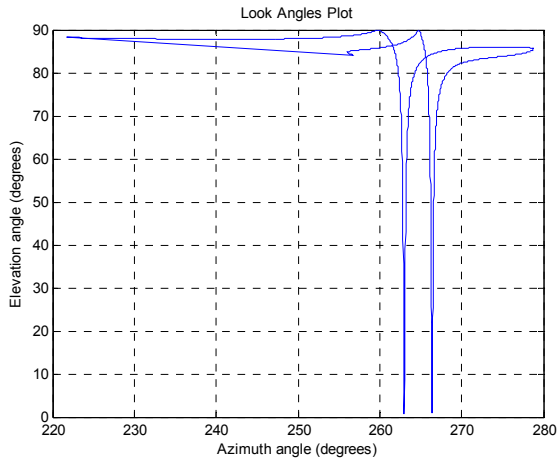


Figure 2: Look Angles plot of Integral

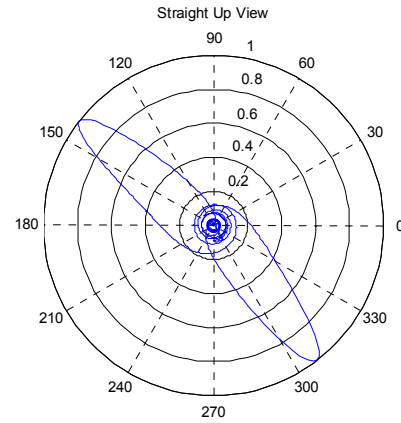


Figure 3: Straight Up View from Van Leer

1.2 Alternate Simulations

In Fig.2, there are coarse jumps and changes in elevation. Fig. 3 seems to descend as if the satellite came crashing to the ground. Since there is clearly an issue with these plots, I decided it was pertinent to verify my results using alternative simulations. I was able to simulate using the heavens-above.com simulator to show a comparison of my own using the same Earth Station (GT Van Leer Building). [4] See the images below.

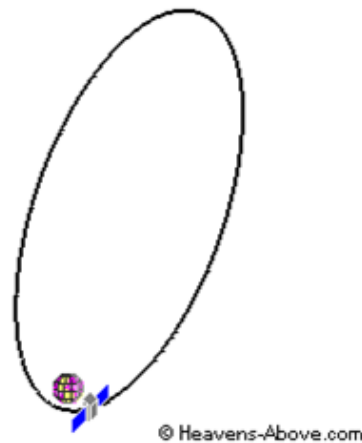


Figure 4: View from above orbital plane

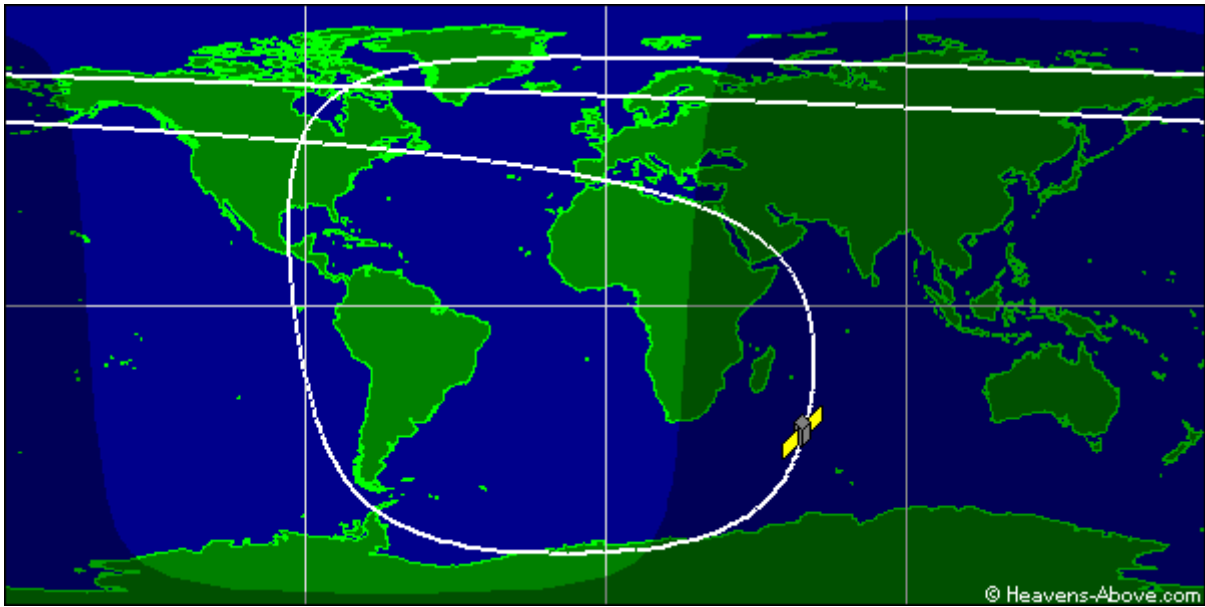


Figure 5: Ground Track

I also utilized the simulation program called JSatTrak to confirm since it has the ability to generate the straight up view that Fig. 3 fails to represent.

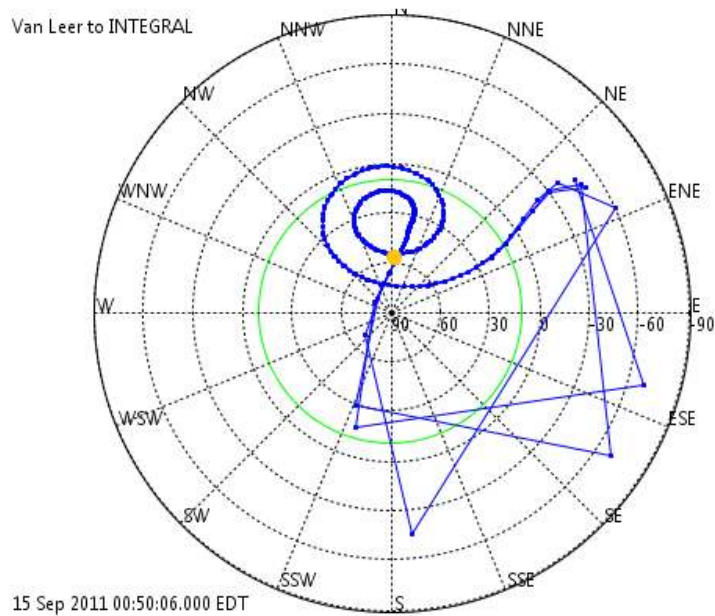


Figure 6: JSatTrak Straight Up View from Van Leer

This seems much more accurate considering the ground track in Fig. 5, although there are wide gaps in the data points in the Eastern to Southern region. However, these seem to be as a result of

the satellite at its perigee (closest point to Earth) and effectively speeding up in its orbit as suggested by Kepler's second law.

2 Application

An application for the satellite could potentially be observing terrestrial anomalies out well beyond 160,000 km away from Earth. In the USC database it is described with the comment, "Astronomical satellite for observing the gamma-ray sky" which suggests that it is more of a scientific vessel than anything else. Also, its purpose is listed as "Space Physics" which affirms this hypothesis. [1] Integral is equipped with an imager and a spectrometer which are supported by an X-ray monitor and an optical camera. Co-aligning these four instruments to the same observable point, scientists are able to identify gamma-ray sources. This is significant since gamma-rays result from some of the highest energy processes in the observable Universe. [4] From reading other sources, it is stated that, "Integral is the first space observatory that can simultaneously observe objects in gamma rays, X-rays and visible light. Its principal targets are violent explosions known as gamma-ray bursts, powerful phenomena such as supernova explosions, and regions in the Universe thought to contain black holes." [4]

3 References

- [1] 01 May 2011. *UCS Satellite Database*. Database. 8 Sep 2011. <http://www.ucsusa.org/nuclear_weapons_and_global_security/space_weapons/technical_issues/ucs-satellite-database.html>.
- [2] "FAQs: Two-Line Element Set Format" *CelesTrak*. Dr. T.S. Kelso, 2 Aug. 2006. Web. 12 Sept. 2011. <<http://celestrak.com/columns/v04n03/>>.
- [3] *CelesTrak*. Dr. T.S. Kelso, 12 Sept. 2011. Web. 12 Sept. 2011. <<http://celestrak.com/NORAD/elements/science.txt>>.
- [4] "ESA - Integral - Overview." *ESA Communications Portal*. Web. 12 Sept. 2011. <http://www.esa.int/esaMI/Integral/SEME2V0XDYD_0.html>.

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%% Satellite Communication and Navigation Systems Homework 1 M-File
% Written by: Mason Nixon
% Date: 09/08/2011
% Version: 1.0
% Desc: Plot the 2D orbit of a satellite, then translate into 3-space and
% determine the look angles for each point in the orbit.
%% HW1
% 2D Orbit

clear all
close all

% Constants
G = 6.672e-11; % Gravitational constant (Nm^2/kg^2)
Me = 5.974e24; % Earth's mass (kg)
Mp = Me; % Mass of the planet (kg)

% Variables
iter = 24e3; % time (s)
deltaT = 15; % Time duration (s)
r(1) = 159565e3; % Initial Radius (meters) (Assume perigee)
theta = 0;%linspace(0,360,t); % Angle (degrees)
Vr = 0.0; % Radial velocity (km/s)
Vtheta = 5.0e3; % Tangential velocity (km/s)

% Calculations
% 1)Orbit
deltaR(1) = Vr*deltaT;
deltaTheta(1) = (Vtheta/r(1))*deltaT;

for n=1:(iter-1)
    r(n+1) = r(n) + deltaR(n);
    theta(n+1) = theta(n) + deltaTheta(n);
    deltaR(n+1) = deltaR(n) + ((r(n)+.5*deltaR(n))*deltaTheta(n)^2 - ((G*Mp)/(r(n)^2))* (deltaT)^2);
    deltaTheta(n+1) = deltaTheta(n) - ((2*deltaR(n)*deltaTheta(n))/(r(n)+.5*deltaR(n)));
end
% 2)Eccentricity and Period
[x,y] = pol2cart(theta,r);

a = 0; % Semi-major axis
b = 0; % Semi-minor axis
eccentricity = sqrt(1-(b/a)^2);

period = sqrt((4*pi^2*a^3)/(G*Mp)); % in seconds

% Plot
figure(1)
polar(theta, r./1000, 'r. ');
title(sprintf('Orbit (r=%1.0fkm, \theta=%1.0f\circ, V_r =%1.1fkm/s, V_{\theta}=%1.1fkm/s)
Eccentricity, e=%1.3f, Period, T=%2.2fhr', ...
    deltaR/1000,deltaTheta,Vr/1000,Vtheta/1000,eccentricity,period/60^2));
view(90,270); %by default the view is (0,90)

%% 3D Orbit Conversion
Inclination = 74.1108; % (in degrees)
Inclination = Inclination*(pi/180); % (convert to radians)

% 3D coordinates
xPrime = deltaR .* cos(deltaTheta);
yPrime = deltaR .* sin(deltaTheta);
zPrime = 0; % Normalize elevation to 0 for now

% Inclination
x = (xPrime.*cos(Inclination) - zPrime.*sin(Inclination));
y = yPrime;
z = (xPrime.*sin(Inclination) + zPrime.*cos(Inclination));

% Convert to Latitude/Longitude
R = sqrt(x.^2 + y.^2 + z.^2);

% Earth Rotational Correction

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%VanLeerLat = 33.775959;
%Circe = 25000; % Earth circumference in miles
%speedEarth = 25000/24; % Earth's speed at equator
%speedVanLeer = speedEarth*cosd(VanLeerLat); % Speed of rotation at Van Leer
speedDPH = 360/24; % Earth travels 360 deg of longitude every ~24 hrs
speedDPS = speedDPH/60^2; % Convert to degrees per second
Correction = speedDPS*15; % For every data point 15 seconds has passed (dt=15sec)

% Lat/Lon
Latitude = asin(z./R);
Longitude = atan2(y,x);
Longitude = Longitude.*(180/pi) - Correction; % convert to degrees & correct for Earth rotation

plot(Longitude, Latitude)

%% Look Angles

earthRadius = 6370e3; % in meters

ESLat = 33.7758*(pi/180); % Earth station latitude (radians)
ESLon = 2*pi -84.39738*(pi/180); % Earth station longitude (radians)

SSPLat = Latitude; % Satellite subpoint latitude
SSPLon = Longitude; % Satellite subpoint longitude
satRadius = z; % Distance from satellite to Earth center

for a1 = 1:length(z);
    gamma(a1) =
acosd(sind(SSPLat(a1)).*sind(ESLat)+cosd(SSPLat(a1)).*cosd(ESLat).*cosd(SSPLon(a1)-ESLon));
    Elevation(a1) = acosd(sind(gamma(a1))/sqrt(1+(earthRadius/satRadius(a1))^2-
2*(earthRadius/satRadius(a1))*cosd(gamma(a1))));
    alpha(a1) = asind(sind(abs(ESLon-SSPLon(a1))).*(cosd(SSPLat(a1))./sind(gamma(a1))));

    % Azimuth adjustment
    % SSP is SW of ES Az=180+alpha
    if(SSPLon(a1) < ESLon & SSPLat(a1) < ESLat)
        Azimuth(a1) = 180 + alpha(a1);
    % SSP is SE of ES Az=180-alpha
    elseif(SSPLon(a1) >= ESLon & SSPLat(a1) < ESLat)
        Azimuth(a1) = 180 - alpha(a1);
    % SSP is NW of ES Az=360-alpha
    elseif(SSPLon(a1) < ESLon & SSPLat(a1) >= ESLat)
        Azimuth(a1) = 360 - alpha(a1);
    % SSP is NE of ES Az=alpha
    elseif(SSPLon(a1) >= ESLon & SSPLat(a1) >= ESLat)
        Azimuth(a1) = alpha(a1);
    end
end

plot(Azimuth, Elevation)
xlabel('Azimuth angle (degrees)');
ylabel('Elevation angle (degrees)');
title('Look angles plot');
grid on;

```