The Georgia Institute of Technology

ECE 6390 Satellite Communication and Navigation Systems
Fall 2011 Homework Assignment 2

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Assignment

In class, we worked through an analytical example using acoustic diffraction theory to find the radiation pattern due to a circular aperture of radius R with uniform field illumination. In this homework problem, you will explore how the patterns of dish antennas change as the illumination changes. Solution will require a relatively straight-forward numerical evaluation of the area integration we derived in class. You may use any software package or technique available to you (be sure to attach your code/inputs).

1 Solution

Using the Kirchoff-Helmholtz Integral Theorem for far-field evaluation:

$$\tilde{E}_{\diamond}(r,\theta,\varphi) = \frac{jkcos(\theta)}{4\pi r} e^{-jkr} \int_0^R \int_0^{2\pi} E_{\diamond}(\rho') e^{jksin(\theta)\rho'\cos(\varphi')} d\varphi' \rho' d\rho'$$

The electric field illumination that we will use across the circular aperture will take the form:

$$E_{\diamond}(\rho') = E_o \cos^n\left(\frac{\pi\rho'}{R}\right), for\ |\rho| \leq R,$$

where E_o is an arbitrary constant and n is the unitless “aperture taper factor” (n=0, of course, corresponds to uniform illumination).

$$\tilde{E}_{\diamond}(r,\theta,\varphi) = \frac{jkcos(\theta)}{4\pi r} e^{-jkr} \int_0^R \int_0^{2\pi} E_o \cos^n\left(\frac{\pi\rho'}{R}\right) e^{jksin(\theta)\rho'\cos(\varphi')} d\varphi'\rho' d\rho' =$$

Note: $J_n(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{(-j(n\tau - xsin\tau))} d\tau$

We evaluate the integral and obtain the 0th order Bessel function:

$$= \frac{jkcos(\theta)}{4\pi r} e^{-jkr} E_o \int_0^R \cos^n\left(\frac{\pi\rho'}{R}\right) 2\pi J_0(ksin(\theta)\rho')\rho' d\rho' =$$

Note: The wave number, $k = \frac{2\pi}{\lambda}$

$$= \frac{j\pi cos(\theta)}{\lambda r} e^{-jkr} E_o \int_0^R \cos^n\left(\frac{\pi\rho'}{R}\right) J_0\left(\frac{2\pi\sin(\theta)\rho'}{\lambda}\right) p' d\rho' =$$

$$\text{Directivity} = D(\theta,\varphi) = \frac{\|\vec{S}\|}{\|\vec{S}_{iso}\|} = \frac{|\tilde{E}^2|}{|\tilde{E}^2_{iso}|} = \frac{4r^2|\tilde{E}^2|}{E_0 R^2}$$

$$\|\vec{S}\| = \frac{1}{2} Re\{ExH^*\} = \frac{\hat{z}|E_0^2|}{2\eta_0}$$

$$D(\theta,\varphi) = \frac{k^2\cos^2\theta}{R^2}\left[\int_0^R \cos^n\left(\frac{\pi\rho'}{R}\right) J_0(ksin\theta\rho')p' d\rho'\right]^2$$

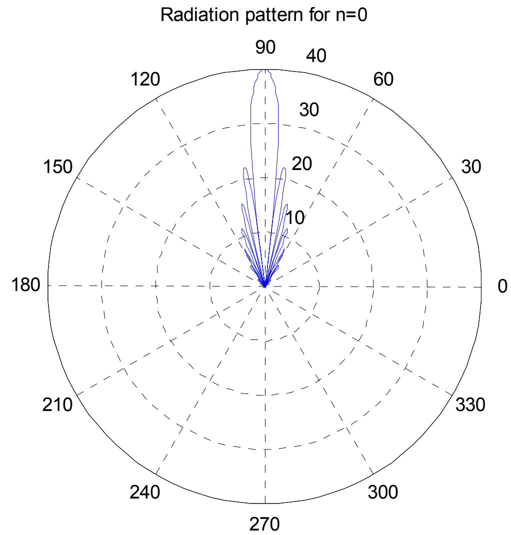
1.1 Simulation

```
% ECE 6390 HW #2 Code
% 27 October 2011
% Modified by Andrew Punnoose and Mason Nixon
% from code by Prof. Durgin

clear all
close all
clc

for n = 0:3      % aperture taper factor
    R = 5;       % dish radius in wavelengths (unitless)
    M = 500;     % number of points to plot
    d = .01;     % numerical integration step size
    Gmin = -40;  % minimum plotting level

    theta = linspace(-pi/2,pi/2,M);
    rhop = 0:d:R;
    D = [];
```

```
    for th = theta
        func = (cos(pi.*rhop./R).^n).*besselj(0,2.*pi.*sin(th).*rhop).*rhop;
        drhop = rhop(2)-rhop(1);
        integral = sum(func).*drhop;
        D = [D,integral];
    end
    Dir = (4.*pi.^2.*(abs(D).^2).*(cos(theta).^2))./(R.^2);
    DdB = 10*log10(Dir);
    DdB = max(DdB - max(DdB) - Gmin, 0);

    figure();
    polar(theta+pi/2, DdB);
    t = sprintf('Radiation pattern for n=%1.0f',n);
    title(t)

    Gpeak = max(DdB);                  % peak gain computation
    hpbw = sum(DdB(1:M) >= Gpeak/2)*pi/M;        % calculate half-power BW
    ind = DdB(2:(M/2))>DdB(1:(M/2-1));             % find local maxima in
gain
    SLL = max(DdB(([0 ind] - [ind 1])>0))/Gpeak; % ... and use for SLL

    % Display results
    fprintf('\n\n Uniform dish of %1.1f wavelengths', R);
    fprintf('\n -----------------------------------');
    fprintf('\n Peak gain:              %3.1f (%2.1f dBi)', ...
        Gpeak, 10*log10(Gpeak) );
    fprintf('\n Half-power beadwidth:   %2.1f deg', hpbw*180/pi );
    fprintf('\n Side-lobe level:        %2.1f dB', -10*log10(SLL) );
    fprintf('\n\n');
end
```

2 Results

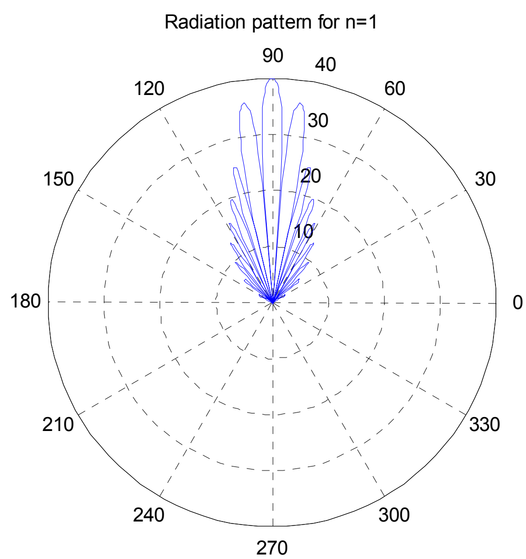
The following results were obtained for a radius of R=5λ and aperture taper factors of n=0,1,2,3 respectively:



Peak gain: 40.0 (16.0 dBi)

Half-power beadwidth: 17.3 deg

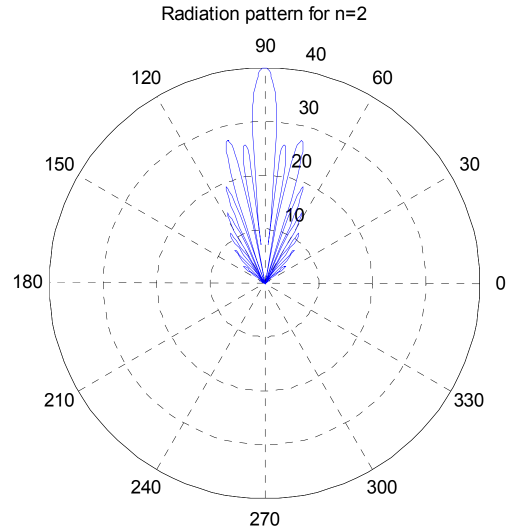
Side-lobe level: 2.5 dB



Peak gain: 40.0 (16.0 dBi)

Half-power beadwidth: 31.0 deg

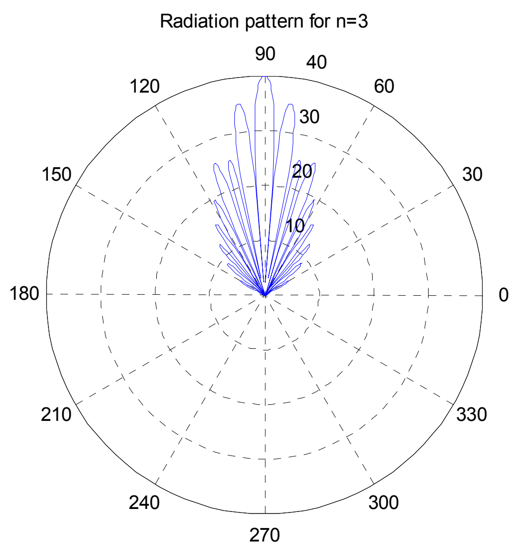
Side-lobe level: 0.5 dB



Peak gain: 40.0 (16.0 dBi)

Half-power beadwidth: 28.8 deg

Side-lobe level: 1.7 dB

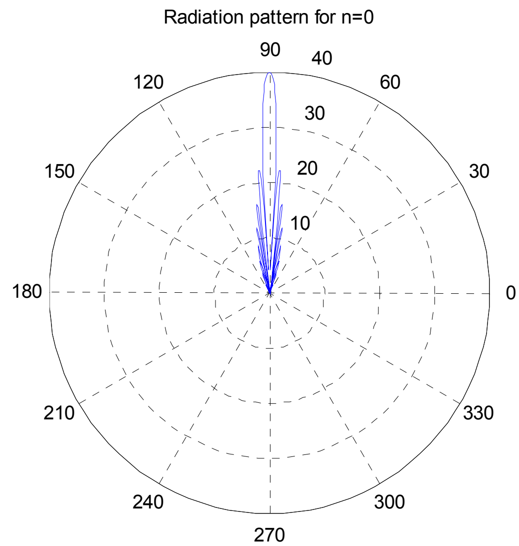


Peak gain: 40.0 (16.0 dBi)

Half-power beadwidth: 38.2 deg

Side-lobe level: 0.6 dB

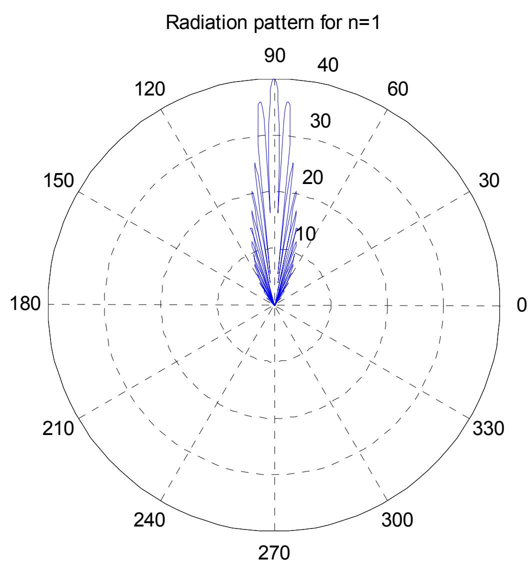
The following results were obtained for a radius of R=10λ and aperture taper factors of n=0,1,2,3 respectively:



Peak gain: 40.0 (16.0 dBi)

Half-power beadwidth: 9.4 deg

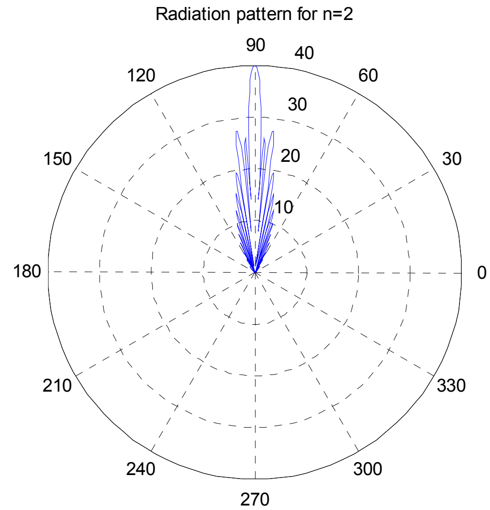
Side-lobe level: 2.5 dB



Peak gain: 40.0 (16.0 dBi)

Half-power beadwidth: 15.8 deg

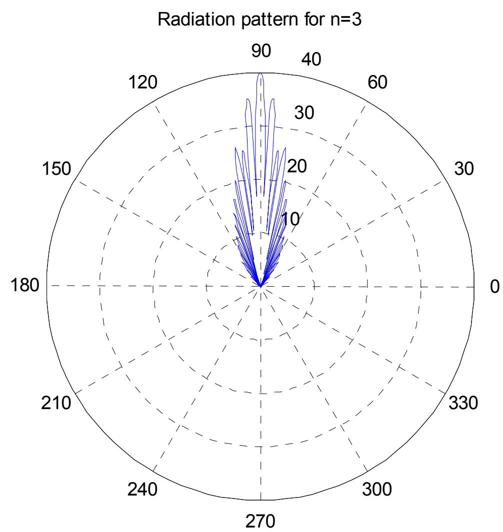
Side-lobe level: 0.4 dB



Peak gain: 40.0 (16.0 dBi)

Half-power beadwidth: 14.4 deg

Side-lobe level: 1.6 dB



Peak gain: 40.0 (16.0 dBi)

Half-power beadwidth: 20.2 deg

Side-lobe level: 0.5 dB

The results show that the odd aperture taper factors result in a larger half-power beamwidth and larger side levels than in the even aperture taper factors. Additionally, for an aperture radius of 10**λ**, we see a narrower main lobe, indicating a more focused beam. This seems intuitive since as we increase aperture size, we get a more focused beam pattern.